Linear Array Partitioning With Minimum Number of Subarrays: An Integer Programming Approach

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***Abstract*—This letter presents an integer programming framework for partitioning a linear array into contiguous, nonoverlapping subarrays while minimizing their total number. The proposed method transforms the partitioning task into a tractable integer programming problem by introducing binary selection variables for candidate subarray configurations. Unlike existing techniques, this formulation does not require predefined subarray counts or reference patterns, offering greater design flexibility. The framework supports uniform excitation for high directivity as well as window-tapered excitation for enhanced sidelobe suppression. Moreover, it can be readily extended to limited-scanning scenarios. Numerical simulations demonstrate that the proposed approach achieves superior performance in directive gain, scan range, and sidelobe control compared to state-of-the-art methods. The method is scalable to large arrays, where high-quality solutions can be efficiently obtained within acceptable time using off-the-shelf solvers.**

***Index Terms*—Integer programming, limited-scanning, linear array, sidelobe level (SLL), subarray partitioning.**

I. Introduction

SUBARRAY partitioning is a fundamental technique for reducing complexity in antenna array systems. The core objective of subarray partitioning is to segment a fully populated antenna array into a smaller number of contiguous, nonoverlapping groups of elements. In this architecture, all elements within a subarray share common radio frequency chains and data converters. Therefore, minimizing the total number of subarrays directly dictates the scale and cost of the required hardware [1], [2], [3]. Determining the partition with the minimum number of subarrays is important for enabling practical and efficient phased array systems in radar and wireless communications [4], [5], [6].

While subarray partitioning effectively reduces system complexity, it inevitably introduces performance tradeoffs, including the increased grating lobes, limited scanning capability, and degraded directivity [7], [8], [9]. Over the past years, numerous partitioning strategies have been proposed to balance these competing performance metrics against hardware simplification. For example, to address the performance degradation associated with a reduced subarray count, partially overlapped subarray configurations have been explored in [1], [6], [10], [11], [12], [13], and [14]. However, a significant drawback of this approach lies in its implementation complexity, stemming from interleaved feed networks and stringent element-spacing requirements. Besides, a range of algorithms based on the K-Means method (KMM) have been proposed [15], [16], [17], [18] to design overlapped subarrays. However, their flexibility is inherently constrained by the prerequisite of predefined reference excitations and radiation patterns. For nonoverlapping partitioning, the compressive sensing (CS) method has been applied to partition arrays with a reduced number of subarrays while jointly optimizing amplitude-phase excitations [19], [20]. However, this approach is sensitive to initialization and may converge to a local optimum. Avser et al. [21] introduced a random search (RS) method for synthesizing partitioned arrays with predetermined subarray lengths and counts. Note that this approach imposes strict constraints on the subarray configuration, which significantly limits the available degrees of freedom in the design process. Beyond the aforementioned strategies, several techniques are proposed to partition subarrays by utilizing the total variation (TV) norm optimization method [22], [23], graph theory [24], [25], or employing two-stage convex optimization (CO) [26], [27], [28]. In addition, some methods focus on array partitioning with fixed-scale subarrays [29], [30].

In this letter, we propose an integer programming approach for partitioning linear arrays into contiguous, nonoverlapping subarrays with the minimum possible count. The proposed formulation provides design flexibility, enabling the use of different window tapering functions for performance enhancements. Moreover, the model can be efficiently solved using standard integer programming solvers. Different from the methods in [29] and [30] that are limited to fixed-scale subarrays, our method supports multiscale subarray partitioning. Simulation results demonstrate that the proposed approach outperforms existing methods in key performance metrics, including directivity and usable scanning range. In addition, the proposed method can be executed within an acceptable time, providing a computational time advantage over the CO algorithm [27], [28], particularly in large-scale array scenarios.

II. Linear Array Partitioning With Minimum Number of Subarrays

A. Preliminaries and Problem Description

To elucidate the problem, a 1-D linear array with N antennas is taken as an example. Under the far-field narrowband assumption, the radiation pattern of the array can be written as $f(\theta) = |\mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta)|$, where $\mathbf{w} = [w_1, \dots, w_N]^{\mathrm{T}}$ is the excitation vector with complex values, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ represent transpose operation and Hermitian transpose operation, respectively. The

Received 3 February 2026; revised 21 March 2026; accepted 30 March 2026. Date of publication 6 April 2026; date of current version 4 June 2026. This work was supported in part by the Sichuan Science and Technology Program under Grant 2024NSFSC1433 and in part by the Peng Cheng Shang Xue Education Fund under Grant XY2021602. *(Corresponding author: Xuejing Zhang.)*

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Digital Object Identifier 10.1109/LAWP.2026.3680928

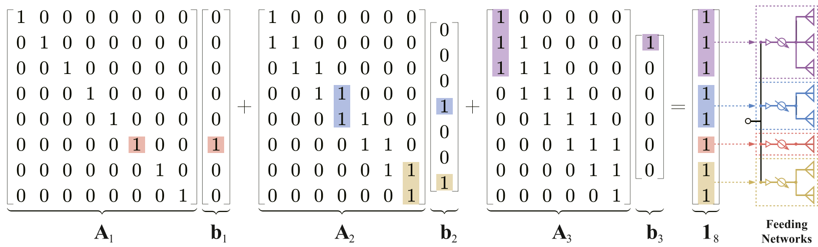

Fig. 1. Schematic of the nonoverlapping partition strategy and its representation via integer programming constraint.

steering vector $\mathbf{a}(\theta)$ is defined as

$$\mathbf{a}(\theta) = \left[g_1(\theta)e^{-j\omega\tau_1(\theta)}, \ldots, g_N(\theta)e^{-j\omega\tau_N(\theta)}\right]^{\mathrm{T}} \quad (1)$$

where ω is the operating frequency, $g_n(\theta)$ denotes the element pattern of the nth antenna, and $\tau_n(\theta)$ is the time delay relative to the reference element, $n = 1, \ldots, N$.

Through subarray partitioning, we can divide N array elements into several groups, with each group sharing the same amplitude and phase excitation. In this article, it is assumed that all elements have uniform feeding power, while the subarray-level phase excitation is determined by the phase center of each subarray. We consider nonoverlapping partitions of adjacent array elements. The right-hand side of Fig. 1 illustrates an example of partitioning a linear array with $N = 8$ elements into four subarrays. It is evident that the number of elements within each subarray may be unequal. To reduce cost and complexity, our objective is to minimize the number of subarray partitions while maintaining satisfactory radiation pattern performance.

B. Subarray Partitioning Using Integer Programming

Before proceeding further, let $\mathbb{K}$ denote the set of all possible numbers of elements that a subarray may contain. The set $\mathbb{K}$ is typically determined based on hardware constraints. Then, for each $k \in \mathbb{K}$, we define the following $N \times (N - k + 1)$ matrix $\mathbf{A}_k$:

$$\mathbf{A}_k = [\mathbf{e}_1^{(k)}, \mathbf{e}_2^{(k)}, \ldots, \mathbf{e}_{N-k+1}^{(k)}]$$
$$= \begin{bmatrix} 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 1 & 1 & 0 & \cdots & 0 & 0 \\ & & \ddots & & \ddots & & \ddots & & \\ 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 1 & 1 \end{bmatrix}^{\mathrm{T}} \quad (2)$$

where $\mathbf{e}_i^{(k)}$ denotes an N-dimensional constant vector with the ith to the $(i + k - 1)$th elements being ones while others being zeros. Each column of $\mathbf{A}_k$ represents a given subarray configuration, with value 1 indicating that the corresponding elements are divided into the same subarray.

To facilitate the subarray partitioning, we introduce a group of binary vectors $\mathbf{b}_k \in \{0, 1\}^{N-k+1}$ to select among possible subarray configurations, where $k \in \mathbb{K}$. For nonoverlapping subarray partitions, it leads to the following constraint:

$$\sum_{k \in \mathbb{K}} \mathbf{A}_k \mathbf{b}_k = \mathbf{1}_N \quad (3)$$

where $\mathbf{1}_N$ represents the N-dimensional all-one vector. In the constraint (3), a value of 1 in $\mathbf{b}_k$ selects the corresponding column of $\mathbf{A}_k$, where each column represents a specific subarray configuration and equation constraint enforces that each antenna is assigned to a single subarray guaranteeing a nonoverlapping partition, and partially overlapped subarray can be achieved by

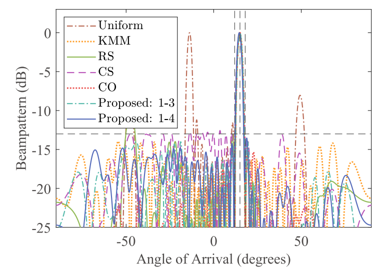

Fig. 2. Focused-beam comparison of different methods in the scenario of subarray partitioning.

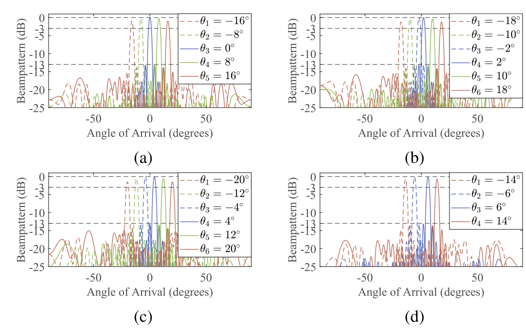

Fig. 3. Scanning beampattern of the array synthesized by the proposed method. (a) Beams scan to 0°, ±8°, and ±16°. (b) Beams scan to ±2°, ±10°, and ±12°. (c) Beams scan to ±4°, ±12°, and ±20°. (d) Beams scan to ±6° and ±14°.

replacing the equal sign with a semipositive definite symbol. Evidently, the partition is uniquely determined by vector set $\{\mathbf{b}_k\}_{k \in \mathbb{K}}$; the number of subarrays is simply the count of elements equal to 1 in $\{\mathbf{b}_k\}_{k \in \mathbb{K}}$. For illustration, Fig. 1 shows a schematic of a specific subarray partition for the case of $N = 8$ and $\mathbb{K} = \{1, 2, 3\}$. By appropriately setting the values of $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$, we can achieve a partition of the array into four subarrays. The number of elements in each subarray can be described by the sequence $\mathcal{S} = (3, 2, 1, 2)$.

With the notation established above, we proceed to model the excitation vector. Take focused-beam synthesis as an example and assume that the beam is steered to θ_0. For the case of equal element power feeding, and given that a common excitation is shared within each subarray, we can prederive the following matrix:

$$\mathbf{Z}_k = \mathbf{A}_k \mathrm{diag}\{\mathbf{z}_k\} \quad (4)$$

where the operator $\mathrm{diag}\{\cdot\}$ constructs a diagonal matrix with diagonal entries being given by the elements of the input vector. The vector $\mathbf{z}_k$ in (4) is a predefined and can be computed as

$$\mathbf{z}_k = \left[e^{j\angle\beta_1^{(k)}}, \ldots, e^{j\angle\beta_{N-k+1}^{(k)}}\right]^{\mathrm{T}} \quad (5)$$

with $\beta_i^{(k)}$ representing the ith entry of the vector $\mathbf{A}_k^{\mathrm{T}}\mathbf{a}(\theta_0)$, $i = 1, \ldots, N - k + 1$. Then, it is not difficult to find that the

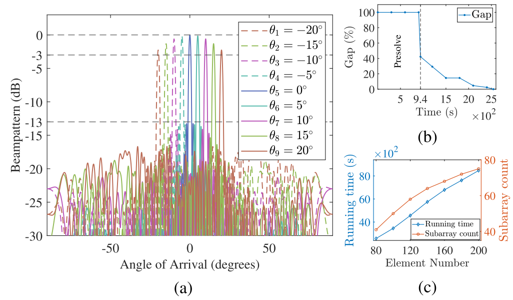

Fig. 4. Results of limited-scanning simulation for large-scaled array. (a) Scanning beampattern. (b) Gap curve of optimization under $N = 80$. (c) Relation between element number and running time, subarray count.

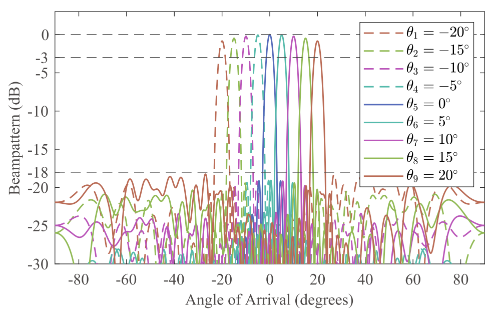

Fig. 5. Scanning beampattern of synthesized under Chebyshev windowing.

element-level excitation vector $\mathbf{w}$ can be expressed as

$$\mathbf{w} = \sum_{k \in \mathbb{K}} \mathbf{Z}_k \mathbf{b}_k. \tag{6}$$

We can now formulate the optimization model for minimizing the number of subarrays as follows:

$$\min_{\{\mathbf{b}_k\}_{k \in \mathbb{K}}} \sum_{k \in \mathbb{K}} \mathbf{1}^{\mathrm{T}} \mathbf{b}_k \tag{7a}$$

$$\text{s.t. } |\mathbf{w}^{\mathrm{H}} \mathbf{a}(\theta_s)| \leq \rho(\theta_s) \cdot \Re[\mathbf{w}^{\mathrm{H}} \mathbf{a}(\theta_0)], \quad \theta_s \in \Theta_s \tag{7b}$$

$$\mathbf{w} = \sum_{k \in \mathbb{K}} \mathbf{Z}_k \mathbf{b}_k \tag{7c}$$

$$\sum_{k \in \mathbb{K}} \mathbf{A}_k \mathbf{b}_k = \mathbf{1}_N \tag{7d}$$

$$\mathbf{b}_k \in \{0, 1\}^{N-k+1}, \quad k \in \mathbb{K} \tag{7e}$$

where $\Re(\cdot)$ represents the operation of taking the real part, $\rho(\theta_s)$ is peak sidelobe level (PSL) preset in the sidelobe region Θ_s in which $\rho(\theta_s)$ is typically determined based on the beamforming design requirements of the task. In the model above, vectors $\{\mathbf{b}_k\}_{k \in \mathbb{K}}$ are binary variables, and $\mathbf{w}$ is implicitly determined by $\{\mathbf{b}_k\}_{k \in \mathbb{K}}$ via constraint (7c). Hence, model (7) is an integer programming model. Given that all constraints are tractable, the above model can be solved efficiently using off-the-shelf integer programming solvers, such as Gurobi [31] and Cplex [32].

Remark 1: While uniform feeding power improves power amplifier efficiency, it may degrade other parameters, such as PSL. This can be mitigated by applying subarray-level windowing to achieve the desired beam characteristics. Specifically, the matrix $\mathbf{Z}_k$ can be modified to $\mathbf{Z}_k = \mathbf{A}_k \mathrm{diag}\{\mathbf{h}_k \odot \mathbf{z}_k\}$, where $\mathbf{h}_k$ denotes a predetermined window function (e.g., a Chebyshev or Taylor window) [21], and $\odot$ represents the Hadamard product. With this modification, the formulation of model (7) remains applicable. However, compared with uniform power weighting, effective aperture shrinkage occurs, which results in mainlobe broadening and a loss in directivity.

TABLE I
RESULTS OF DIFFERENT METHODS UNDER SLL CONSTRAINT

	Proposed		RS	CS	CO	KMM	Uniform
Max subarray length	3	4	3	5	2	3	4
Element count	40	40	40	40	40	40	40
Subarray count	19	18	18	13	20	21	10
SLL (dB)	−14	−13	−10	−13	−15	−14	-
Directivity (dBi)	14.55	14.75	14.92	12.67	14.86	14.46	12.65
Running time (s)	52.74	103.4	4.73	25.17	131.7	0.41	-

Remark 2: In this work, excitation design and subarray partitioning are jointly addressed in a unified model by exploiting a windowing-function strategy. Compared with the two-stage CO methods in [27] and [28], the proposed approach achieves better computational efficiency and convergence, especially for large-scale array problems. Moreover, the proposed framework supports multiscale subarray partitioning, offering higher flexibility than fixed-scale partitioning [29], [30] with only a modest increase in computational cost.

C. Extension to Limited-Scanning Scenario

The aforementioned integer programming approach remains applicable for limited-scanning scenario. Given a set of Q scanning beams, the excitation vector for each beam can be derived following the same form as (6):

$$\mathbf{w}_q = \sum_{k \in \mathbb{K}} \mathbf{Z}_{q,k} \mathbf{b}_k \quad \forall q \in \mathbb{Q} \tag{8}$$

where $\mathbb{Q} \triangleq \{1, 2, \ldots, Q\}$. For a given index k, $\mathbf{Z}_{q,k}$ represents the corresponding excitation matrix determined by (4) steering to qth direction serving a similar role to $\mathbf{Z}_k$ in (6).

Following (7), we can then formulate the subarray partitioning model for limited-scanning scenario as follows:

$$\min_{\{\mathbf{b}_k\}_{k \in \mathbb{K}}} \sum_{k \in \mathbb{K}} \mathbf{1}^{\mathrm{T}} \mathbf{b}_k \tag{9a}$$

$$\text{s.t. } |\mathbf{w}_q^{\mathrm{H}} \mathbf{a}(\theta_s)| \leq \rho_q(\theta_s) \cdot \Re[\mathbf{w}_q^{\mathrm{H}} \mathbf{a}(\theta_q)], \theta_s ! \in ! \Theta_{s,q} \, \forall q \in \mathbb{Q} \tag{9b}$$

$$\mathbf{w}_q = \sum_{k \in \mathbb{K}} \mathbf{Z}_{q,k} \mathbf{b}_k \quad \forall q \in \mathbb{Q} \tag{9c}$$

$$\sum_{k \in \mathbb{K}} \mathbf{A}_k \mathbf{b}_k = \mathbf{1}_N \tag{9d}$$

$$\mathbf{b}_k \in \{0, 1\}^{N-k+1}, \quad k \in \mathbb{K} \tag{9e}$$

where θ_q is the beam axis for the qth scanning, $\rho_q(\theta_s)$ is the corresponding PSL preset in the sidelobe region $\Theta_{s,q}$. Similarly, the above model is tractable and can be solved using off-the-shelf integer programming solvers.

III. NUMERICAL RESULTS

In this section, we set up simulations to verify the performance of the proposed method.[1] For comprehensive performance evaluation, we compare against several state-of-the-art approaches applicable to various scenarios. These include the KMM in [18], CS method in [19], CO method in [28], and RS method in [21]. Among them, KMM and CO are methods partitioning overlapped subarrays. It should be noted that, both excitation phases

[1]The MATLAB codes for the proposed method are available online at https://zhangxuejing7.github.io/HomePage/.

TABLE II
RESULTS OF DIFFERENT METHODS IN THE LIMITED-SCANNING SCENARIO

	Proposed						RS		CS
	Spatial matched		Chebyshev windowing				Spatial matched	Chebyshev windowing	
Max scanning angle	±20°	±20°	±24°	±22°	±20°	±15°	±10°	±18°	±24°
Scanning step	5°	2°	5°	5°	5°	5°	5°	5°	5°
Subarray count	23	23	23	27	27	28	23	27	21
SLL (dB)	−13	−13	−13	−16	−18	−20	−13	−18	−13
Directivity of central beam (dBi)	16.02	16.02	15.83	15.78	15.79	15.73	15.97	15.73	14.03
Directivity of outmost beam (dBi)	14.34	14.33	13.99	14.62	14.95	15.20	15.50	14.99	12.16
Running time (s)	1317	1333	1953	3539	3840	429	757	1144	650

TABLE III
RESULTS OBTAINED WITH VARIOUS $K_{\max}$ AND N

	$K_{\max}=3$			$K_{\max}=5$		
N	40	80	120	40	80	120
HPBW	3.02°	1.51°	0.96°	3.05°	1.57°	0.98°
Directivity (dBi)	14.95	17.01	17.89	14.87	16.91	17.75

and amplitudes of subarrays synthesized by the CS method are considered. Besides, RS method cannot determine the number and length of subarrays adaptively, preset partitioning scheme may well be a suboptimized pattern. We employ uniform linear arrays that consist of N isotropic elements with half-wavelength interelement spacing as full array for partitioning. All simulations are carried in MATLAB R2022a on a PC with Intel Core i7-12700 K CPU 3.60 GHz and a 32.0 GB RAM. The proposed models are solved by Gurobi [31].

A. Linear Array Partitioning Performance of the Proposed Method With Minimum Subarray Count

In the first example, simulations are implemented to demonstrate that the proposed method has an effective performance in partitioning a linear array into subarrays with minimum count and beam synthesis with spatial matched excitations.

1) Single Focused-Beam Synthesis With Minimum Subarray Count: In this scenario, we set $N = 40$. The mainlobe of focused-beam is steered to 10°, and the sidelobe level (SLL) within region $[-90^\circ, 12^\circ] \cup [18^\circ, 90^\circ]$ is constrained below -13 dB or -14 dB. In Fig. 2, beampatterns synthesized by different methods are presented for comparison together with a pattern calculated based on a uniformly separated array in which subarray length is equal to 4 with spatial matched weight. The result of antenna arrays partitioning obtained by the proposed method is 19 subarrays as $\mathcal{S} = (3,3,2,2,1,2,2,2,2,2,2,2,2,2,2,3,1,2,3)$ when setting $\mathbb{K} = \{1,2,3\}$, while 18 subarrays as $\mathcal{S} = (1,2,2,2,2,2,2,2,1,2,2,3,2,3,2,4,3,3)$ with $\mathbb{K} = \{1,2,3,4\}$. Compared with other methods, the proposed approach generates a highly desirable beampattern achieving an excellent balance between directivity and SLL under spatial matched excitations owning equal amplitude. The subarray parameters, SLL, directivity, and running time of different methods are listed in Table I.

2) Limited-Scanning Performance Demonstration: To evaluate the scanning performance of the proposed method, a scannable subarray-partitioned array, where $N = 40$ and $\mathbb{K} = \{1,2,3\}$, is designed to test its maximum scanning angle. The scanning radiation patterns are shown in Fig. 3, where the beam is required to steer from -20° to 20° with a step length of 2°. The subarray segmentation result is $\mathcal{S} = (3,1,2,1,1,2,1,2,2,2,1,2,1,2,1,2,3,2,1,3,2,1,1)$ of which the count is 23. As can be observed from the results, the array synthesized by the proposed method exhibits a relatively wide scanning range, and the mainlobe gain attenuation is less than 3 dB compared with the central beam. This simulation takes an acceptable time for nonreal-time use, approximately 1400 s.

3) Limited-Scanning Performance Demonstration for Large-Scaled Array: To further verify the applicability of the proposed method, large-scaled linear arrays whose $N \in \{80,100,120,140,160,180,200\}$ are deployed, and we set $\mathbb{K} = \{1,2,3,4,5,6\}$. In this simulation, the SLL is required to remain below -13 dB. Results of this simulation are shown in Fig. 4. Take the case of $N = 80$ as an example, the obtained beampattern scanning from -20° to 20° and gap curve are depicted in Fig. 4(a) and (b), respectively, where parameter gap refers to the relative difference between the currently found solution and the theoretical optimum. The ideal radiation pattern and optimizing behavior remain sufficiently demonstrated when addressing complex problems, such as large-scale array scanning. To further verify the efficiency of the proposed method, the time consumption and the resulted subarray counts for large-scale arrays with different Ns are displayed in Fig. 4(c). The observed computational cost illustrates high practice for deployment.

B. Extended Example With Tapering Weights

In this example, Chebyshev window tapering is applied with sidelobe attenuation of 16 dB, 18 dB, 20 dB are considered and we set $N = 40$ and $\mathbb{K} = \{1,2,3\}$. Reconfigurable subarray-partitioned arrays are designed to evaluate the scanning performance under this scenario, and the results of 18 dB attenuation are shown in Fig. 5, where the main beam scans from -20° to 20° with a step size of 5° maintaining an SLL under -18 dB. The grouping configuration result is $\mathcal{S} = (3,2,1,2,2,1,1,1,2,1,1,2,1,2,1,1,1,1,2,2,1,1,2,1,1,2,2)$, yielding a total of 27 subarrays. The parameters obtained under all three attenuation factors containing maximum scanning angles, SLL, directivity of central and outmost beam, running time, as well as parameters under the RS and CS methods are listed in Table II. From Table III, it can be observed that half-power beamwidth (HPBW) is broadened by a larger $K_{\max}$ while the directivity is decreased slightly. Overall, the proposed method is able to effectively exploit the properties of the windowing function to synthesize a subarray-partitioned array with ideal performance.

IV. CONCLUSION

In this letter, we have proposed an integer programming approach for partitioning linear arrays into contiguous, nonoverlapping subarrays with the minimum possible count. Different from existing methods that require predefined subarray counts or reference patterns, our method offers greater design freedom. The presented method is inherently flexible, accommodating both uniform and window-tapered excitations and extending naturally to multibeam limited-scanning systems. In addition, the proposed model can be efficiently solved using standard integer programming solvers. Simulation results have demonstrated that the proposed approach outperforms existing methods in directivity, scan range, and SLLs. Future work may investigate extensions to subarray partitioning for planar arrays.

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