Communication

Beampattern Synthesis for Phased Array With Dual-Phase-Shifter Structure

Yangjingzhi Zhuang, Xuejing Zhang, and Zishu He

***Abstract*— This communication proposes a convex optimization algorithm to synthesize desirable beampattern for phased array with the dual-phase-shifter (DPS) structure. In the DPS structure, dual phase shifters are combined into one beamforming weight. Compared with conventional phased array, the moduli of the weight vector are not necessarily fixed constants in the DPS structure but can be changed in a continuous interval. For this reason, we propose a method that turns the beampattern synthesis problem into a convex optimization problem. The method can be readily implemented and efficiently solved using freely accessible routines. We consider how to obtain the weight vector with two beampattern synthesis problems. The first one is to maximize the mainlobe gain (or minimize the mainlobe loss) when the notch level is determined. The second one is to minimize the notch level when the mainlobe gain (or the mainlobe loss) is determined. In both cases, we can get suboptimal solutions. Under different situations, numerical results of simulations confirm the effectiveness of the two algorithms.**

***Index Terms*— Beampattern synthesis, convex optimization, dual-phase-shifter (DPS), phased array.**

I. Introduction

For a radar system, it is always desirable to minimize the power of the transmitted and received noise sources so that a higher signal-to-noise ratio (SNR) can be obtained. This goal can be achieved by synthesizing various desired beampatterns. The phase shifter is an important device and the phase-only control method is particularly meaningful in beampattern synthesis [1]–[3]. In phase-only control, we only need to adjust the excitation phases to complete the target task, while the excitation amplitudes are known and fixed as constants. Phase-only control is much easier than adjusting the excitation amplitudes for many kinds of antennas. Moreover, phase-only control has the advantages of short response time, high efficiency, and so on.

In the past years, quite a number of approaches to phase-only control have been developed. DeFord and Gandhi [4] proposed a numerical search technique based on phase-only control to minimize the maximum sidelobe level for a given array geometry. The work in [5] introduced a method of constrained power synthesis, which yields array patterns belonging to the masks for a set of given masks, and the reconfiguration from any of these patterns to the others was performed by phase-only control. A new method of phase-only antenna beampattern synthesis was presented in [6], which introduces a scaling factor to accurately represent the shape constraints on both the mainlobe and sidelobe regions. A geometric approach with low computational complexity to fast response adjustment with phase-only constraint was described in [7]. However, the limitation of this algorithm is that it can only adjust one-point response as desired and fails to adjust multiple points with phase-only restriction. For the asymmetric radiation patterns, Kadlimatt and Parimi [8] considered an odd phase excitation to synthesize the desired patterns with uniformly linear arrays (ULAs).

With the development of interior point method [9], convex optimization has been applied to beampattern synthesis [10], [11] and phase-only control [12]. A phase-only method with semidefinite relaxation (SDR) technique to generate notches in the beampattern synthesis of a phased-array antenna was described in [13]. With the same theoretical idea, the work in [14] has developed a general procedure for nonconvex beampattern synthesis problems. In order to solve the constraints that the moduli are fixed in the single-phase shifter structure, Cao *et al.* [15] proposed an alternating optimization algorithm based on SDR to solve the relaxed problem. In [16], a new method was proposed to synthesize high-performance beampatterns, with the aid of linear fractional SDR technique and a quasi-convex optimization approach. Note that these methods may not obtain the optimal solution since the original problems have been relaxed and changed. A new convex optimization method, which does not need to relax constraints, has been devised in [17]. The method uses conjugate symmetric beamforming weights so that the upper and nonconvex lower bound constraints on the beampattern can be convex.

Dual phase shifters are combined to represent one beamforming weight in the DPS structure [18]. The DPS structure makes the design of the weight vector flexible. More specifically, in the DPS structure, the moduli of the weight vector are no longer limited to constant magnitudes but can be flexibly changed in a continuous interval, thus greatly improving its flexibility in weight design. In addition, the analysis in [18] and [19] shows that the DPS structure also has certain advantages in phase quantization. In this communication, we propose a method based on the DPS structure, which can synthesize the desired beampatterns by adjusting excitation phases. We consider two different beampattern synthesis problems to obtain weight vectors. The first one is to maximize the mainlobe gain (or minimize the mainlobe loss) when the notch level is determined. The second one is to minimize the notch level when the mainlobe gain (or the mainlobe loss) is determined. Since the modulus of the weight vector can be changed continuously in the DPS structure, we can turn the original problems into convex problems, which can be solved efficiently by readily available solvers (e.g., CVX toolbox). Simulation results show that the method proposed in this communication is effective. Compared with existing methods, the proposed method can achieve the desired performance in most cases, although it may not be the optimal one.

The rest of this communication is organized as follows. The system model is presented in Section II. The proposed DPS method is analyzed in Section III. In Section IV, numerical results verify the performance of the proposed method. Finally, conclusions are drawn in Section V.

II. System Model

A. Beampattern Synthesis

We consider an arbitrary 1-D array with N antennas. The steering vector associated with θ is

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given as

$$\mathbf{a}(\theta) = [g_1(\theta)e^{j\Phi_1}(\theta), \ldots, g_N(\theta)e^{j\Phi_N}(\theta)]^{\mathrm{T}} \tag{1}$$

where θ stands for the incidence angle, $\Phi_n(\theta)$ represents the phase delay of the nth element, $g_n(\theta)$ is the individual pattern for the nth element, and $g_n(\theta) = 1$ when the antenna is isotropic. $[\cdot]^{\mathrm{T}}$ denotes the transpose operator and $j = \sqrt{-1}$ is the imaginary unit. For a given weight vector $\mathbf{w}$, the array output can be expressed as

$$y(\theta) = \mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta). \tag{2}$$

We can synthesize the desired beampattern $|y(\theta)|$ by adjusting the appropriate weight vector $\mathbf{w}$.

B. DPS Structure

It is worth noting that the modulus of each element of $\mathbf{w}$ is different for the traditional phased-array structure and DPS structure. This is due to their different structure. As shown in Fig. 1(a), each element of the traditional phased array contains only one phase shifter, that is, N antenna branches correspond to N phase shifters and N phase shifters correspond to N elements of the weight vector $\widetilde{\mathbf{w}}$. Thus, we know $|\widetilde{w}_i| = 1, i = 1, 2, \ldots, N$, where $\widetilde{w}_i$ represents the ith element of $\widetilde{\mathbf{w}}$, while for the DPS structure as shown in Fig. 1(b), when the number of elements is unchanged, the number of the phase shifters is doubled, and each array element contains two phase shifters, that is, N antenna branches correspond to $2N$ phase shifters and two phase shifters correspond to one element of the weight vector $\mathbf{w}$. Therefore, this weight vector in DPS can be expressed as $\mathbf{w} = \mathbf{w_1} + \mathbf{w_2}$ and $|w_i| \in [0, 1]$,[1] $i = 1, 2, \ldots, N$, where w_i represents the ith element of $\mathbf{w}$, $\mathbf{w_1}$ and $\mathbf{w_2}$ are weight vectors corresponding to the two sets of phase shifters.

Moreover, note that the method proposed in this communication is based on a DPS structure. The weight vector obtained directly is actually the sum of the two weight vectors corresponding to the two sets of phase shifters, so we need to determine the phases of two weight vectors [18] by

$$\phi_i = \angle w_i + \cos^{-1}(|w_i|) \tag{3a}$$
$$\varphi_i = \angle w_i - \cos^{-1}(|w_i|) \tag{3b}$$

with $\phi_i = \angle w_{1,i}$ and $\varphi_i = \angle w_{2,i}$, where $w_{1,i}$ and $w_{2,i}$ denote the ith element of $\mathbf{w_1}$ and $\mathbf{w_2}$, respectively, and $|\cdot|$ and $\angle\cdot$ stand for the operation of returning absolute values and outputting complex phases, respectively.

III. Beampattern Synthesis Method With DPS

In this section, we use the DPS structure and consider two kinds of beampattern synthesis problems.

A. Mainlobe Gain Maximization

In engineering applications, we always hope to obtain the maximum mainlobe gain to guarantee a high output SNR. Meanwhile, the sidelobe levels need to be controlled, and this problem can be expressed as

$$\max_{\mathbf{w}} \ |\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)| \tag{4a}$$
$$\text{s.t.} \ \frac{|\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_s)|}{|\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)|} \le \rho_s \quad \theta_s \in \mathbf{\Omega}_S \tag{4b}$$
$$\|\mathbf{w}\|_\infty \le 1 \tag{4c}$$

[1]In practical implementations, when only passive components are used, the output power will not be higher than the input power. To reflect this fact, it is more appropriate to scale 1/2 to each point after splitting. Thus, the maximum modulus of each element of the weight vector $\mathbf{w}$ is one.

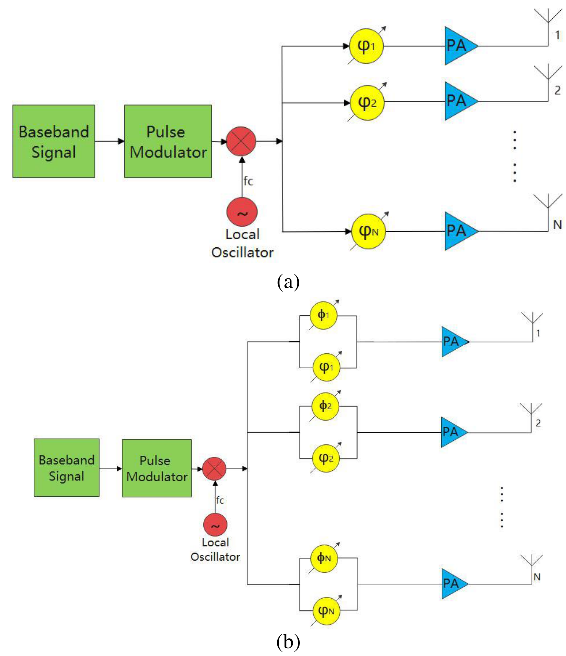

Fig. 1. Illustration of phased-array transmission structure. (a) Traditional phased-array structure. (b) Phased array with DPS structure.

where $\mathbf{w} = [w_1, w_2 \ldots, w_N]^{\mathrm{T}}$, $\|\cdot\|_\infty$ denotes the infinite norm operator, θ_0 stands for the beam axis direction, and ρ_s is the given upper bound level of the prescribed sidelobe region $\mathbf{\Omega}_S$.

However, the problem described in (4) is not a standard convex optimization problem, and it will cause certain difficulties in solving. In order to tackle the problem (4), we introduce an auxiliary variable α, and let $|\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)| = \alpha$. Then, we transform the objective function to find the minimum value of $-\alpha$. Now, we can describe the problem in (4) as follows:

$$\min_{\mathbf{w}} \ -\alpha \tag{5a}$$
$$\text{s.t.} \ |\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)| = \alpha \tag{5b}$$
$$\frac{|\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_s)|}{|\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)|} \le \rho_s \quad \theta_s \in \mathbf{\Omega}_S \tag{5c}$$
$$\|\mathbf{w}\|_\infty \le 1. \tag{5d}$$

Note that (5b) is still nonconvex. The constraints in (5) are unchanged when $\mathbf{w}$ undergoes an arbitrary phase rotation. For this reason, we can always rotate the phase of optimal solution without affecting the objective function value so that $\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)$ is real. Therefore, the problem of (5) can be written as

$$\min_{\mathbf{w}} \ -\alpha \tag{6a}$$
$$\text{s.t.} \ \mathrm{Re}\{\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)\} = \alpha \tag{6b}$$
$$\mathrm{Im}\{\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)\} = 0 \tag{6c}$$
$$\frac{|\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_s)|}{|\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)|} \le \rho_s \quad \theta_s \in \mathbf{\Omega}_S \tag{6d}$$
$$\|\mathbf{w}\|_\infty \le 1. \tag{6e}$$

Since $\mathrm{Re}\{\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)\}$ and $\mathrm{Im}\{\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)\}$ are affine and the constraints in (6d) and (6e) are convex, we can see that problem (6) is a convex optimization problem. After obtaining the optimal weight vector $\mathbf{w}$ through (6), we combine (3) to obtain $\mathbf{w_1}$ and $\mathbf{w_2}$.

B. Notch Level Minimization

Interference is always accompanied by signal transmission. When the signal gain in the mainlobe direction meets the requirements we need, we turn to consider suppressing the interference in the sidelobe direction. The problem can be described as

$$\min_{\mathbf{w}} \ \max|\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_p)| \quad \theta_p \in \mathbf{\Omega}_P \tag{7a}$$
$$\text{s.t. } |\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)| \geq C \tag{7b}$$
$$\|\mathbf{w}\|_\infty \leq 1 \tag{7c}$$

where $\mathbf{\Omega}_P$ is the specific sidelobe region, and C is a constant, which stands for the minimum acceptable gain for a target in the center of the main beam.

Similarly, it is not hard to find that the problem in (7) is not a convex optimization problem. Therefore, we introduce an auxiliary variable t, let $\max|\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_p)| = t$, and rewrite the objective function as the problem of minimizing t. Thus, we can describe (7) as follows:

$$\min_{\mathbf{w}} \ t \tag{8a}$$
$$\text{s.t. } |\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_p)| \leq t \quad \theta_p \in \mathbf{\Omega}_P \tag{8b}$$
$$|\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)| \geq C \tag{8c}$$
$$\|\mathbf{w}\|_\infty \leq 1. \tag{8d}$$

In addition, the constraint on the mainlobe in (8c) is not a convex constraint. Using the similar operation as described in Section III-A, we can remove the absolute value in (8c) and further describe the problem as follows:

$$\min_{\mathbf{w}} \ t \tag{9a}$$
$$\text{s.t. } |\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_p)| \leq t \quad \theta_p \in \mathbf{\Omega}_P \tag{9b}$$
$$\mathrm{Re}\{\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)\} \geq C \tag{9c}$$
$$\mathrm{Im}\{\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)\} = 0 \tag{9d}$$
$$\|\mathbf{w}\|_\infty \leq 1. \tag{9e}$$

Similar to problem (6), we can know that problem (9) is also convex. We combine (3) to obtain $\mathbf{w_1}$ and $\mathbf{w_2}$ after obtaining the optimal weight vector $\mathbf{w}$ through (9).

IV. Simulations

In this section, we carry out representative simulations to validate the proposed method. For the convenience of discussion, we assume that the noise is Gaussian white noise, so the quiescent weight vector equals $\mathbf{a}(\theta_0)$. For comparison purpose, the convex programming (CP) in [12], the phase-only method in [13], and the $\mathrm{A^2RC}$ method in [20] are also tested. To guarantee the significance of the simulation comparison results, we need to ensure that the transmission power of each method is the same.

A. Mainlobe Gain Maximization

In this section, we consider two types of arrays: the ULA and the random array. The mainlobe gain is maximized with a given sidelobe level.

1) Uniform Linear Array: In this example, let us consider a ULA of $N = 32$ isotropic elements spaced by half wavelength. We fix the beam axis as $\theta_0 = 30°$. The desired notch level is expected to be lower than −55 dB at the region of the given notch $\mathbf{\Omega}_S = [-50°, -35°]$. The simulation results are shown in Fig. 2.

From Fig. 2, we can see that the proposed method, CP, and $\mathrm{A^2RC}$ method meet the desired notch level −55 dB at the set sector $\mathbf{\Omega}_S$, and the notch level of the phase-only method is slightly higher than the requirement. We also note that the sidelobe levels of the proposed method are lower than other three methods in some directions. Moreover, the resulting mainlobe losses of the proposed method, phase only, CP, and $\mathrm{A^2RC}$ are 0.1907, 0.2782, 0.0380, and 0.0273 dB, respectively. The mainlobe loss of the proposed method is less than phase-only method but more than CP and $\mathrm{A^2RC}$ method. Since the modulus of the weight vector is still constrained, the performance of the proposed method may perform worse than those methods that have no constraint on the magnitudes, such as CP.

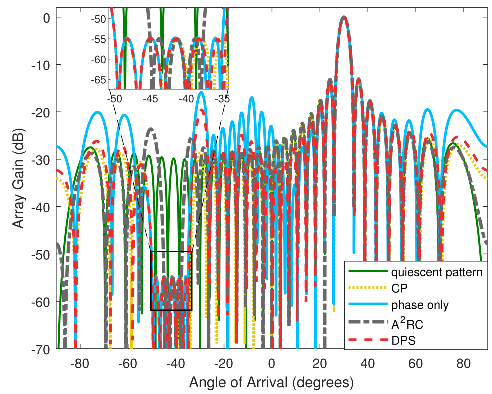

Fig. 2. Beampattern comparison for mainlobe gain maximization with an ULA.

The depth of the notches and the mainlobe gain loss are related, which has been pointed out in [13]. Fig. 3 shows the changes of the mainlobe gain loss under various levels of the notch, which is from −60 to −30 dB, and the other parameters are unchanged. For all the methods tested, we can predict that the higher the notch level takes, the less the mainlobe loss can be resulted. One can readily check from Fig. 3 that the resulting curves are consistent with our prediction. Also, we can see that the resulting mainlobe losses of the proposed method are always less than those of the phase-only method. The minimal mainlobe losses are obtained by the CP method. Moreover, the mainloss approaches zeros with the notch levels approaching −30 dB; −30 dB is the corresponding level of the quiescent pattern in the notch area $\mathbf{\Omega}_S$, which is shown in Fig. 2. This shows that when the weight vectors of all methods degrade to quiescent weight vector, there is no array gain loss. The synthesized beampatterns and quiescent beampattern completely coincide.

2) Random Linear Array: To further examine the performance of the proposed method, we consider a nonuniform linear array that is formed by $N = 32$ nonisotropic elements. The individual pattern for the nth element is expressed as

$$g_n(\theta) = \frac{\cos[\pi l_n \sin(\theta + \zeta_n)] - \cos(\pi l_n)}{\cos(\theta + \zeta_n)} \tag{10}$$

where l_n and ζ_n represent the length and the orientation of the nth element, respectively. Also, the specific data are presented in Table I, where x_n is the position of the nth element. We steer the beam to $\theta_0 = -10°$. The notch region is $\mathbf{\Omega}_S = [-60°, -40°]$, and the notch level is imposed to be lower than −40 dB. From Fig. 2, it can be seen that all the methods fulfill the requirements well. Also, the mainloss of the proposed method is 1.8157 dB, which is the same as that of the CP method. Moreover, the mainloss of the proposed method is less than that of the $\mathrm{A^2RC}$ method (3.7884 dB) and the phase-only method (2.6358 dB). Thus, the validity of the proposed method has been verified again.

B. Notch-Level Minimization

In this section, we also specify the region of the notch on the beampattern first, and the other sidelobe is unconstrained. Then, we need to confirm the gain loss in the mainlobe. To show the mainlobe loss, we introduce a variable $\rho(w) = 20log(|\mathbf{w}^{\mathrm{T}}\mathbf{a}(\theta_0)|/|\mathbf{a}^{\mathrm{T}}(\theta_0)\mathbf{a}(\theta_0)|)$. Two different array structures are set in the following.

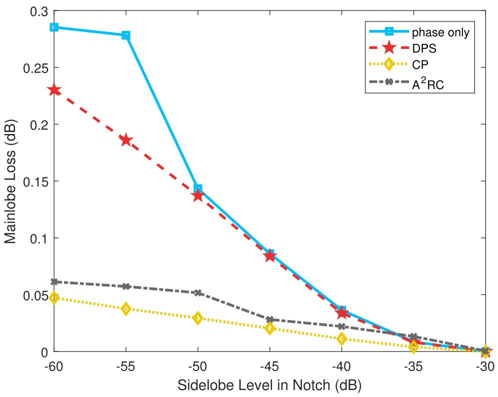

Fig. 3. Curves of mainlobe loss versus the maximum sidelobe level in notch.

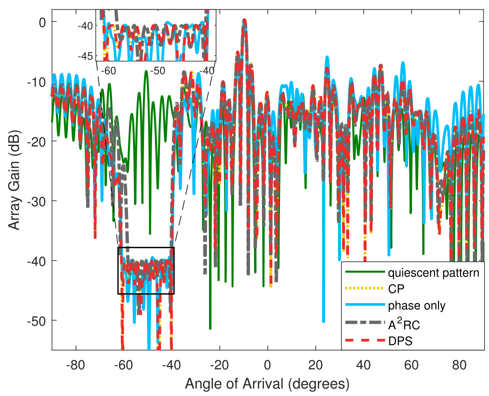

Fig. 4. Beampattern comparison for mainlobe gain maximization with a random linear array.

1) Uniform Linear Array: For illustration, we use a ULA of $N = 32$ elements spaced by half wavelength. We fix the beam axis as $\theta_0 = -50^\circ$, and the notch region is $\Omega_S = [50^\circ, 70^\circ]$. The desired mainlobe loss is set in advance, which is 0.2 dB. The results are shown in Fig. 5.

From Fig. 5, we can see that all the methods fulfill the requirement in the mainlobe and obtain a deep notch in the given notch region. The notch level of the proposed method, phase-only, CP, and A^{2}RC are -53.3697, -50.8693, -63.2074, and -55.8534 dB, respectively. The notch level of the proposed method is slightly below that of the phase-only method. Similar to the situation Section IV-A is that the CP method obtains the best performance. In summary, the effectiveness of the proposed method gets proved, but the performance of the proposed method is worse than that of CP, which does not limit magnitudes.

Similar to above, we also study the relationship between the levels of the notch and the losses in the mainlobe. As the loss of the mainlobe changes, the depth of the notch keeps changing, which is shown in Fig. 6. It can be expected that the level of notch decreases as the mainlobe loss increases for all methods. From Fig. 6, the prediction is verified. Under the same mainlobe loss, the notch level of the proposed method is always lower than that of the phase-only method and is also lower than that of A^{2}RC method twice. Moreover, when the mainlobe loss is getting to zero, the notch level of the four methods is approaching to -25 dB, which is the sidelobe level of the quiescent pattern in the prescribed notch region. It is shown in Fig. 5.

2) Random Linear Array: To further study the proposed method performance, in this section, we consider the nonuniform linear array of the second example of Section IV-A. In this case, we set the beam axis as $\theta_0 = 10^\circ$, and the notch region is $\Omega_S = [-60^\circ, -40^\circ]$.

TABLE I

PARAMETERS OF THE RANDOM ARRAY AND THE OBTAINED WEIGHTINGS BY THE PROPOSED METHOD

n	$x_n(\lambda)$	$l_n(\lambda)$	ζ_n(deg)	w_n
1	0.00	0.27	0.0	$0.4982e^{-j1.6250}$
2	0.275	0.26	0.5	$0.7766e^{+j3.0738}$
3	0.775	0.21	5.0	$1.0000e^{-j0.3541}$
4	1.025	0.21	-32	$0.7363e^{-j1.4512}$
5	2.525	0.22	-3.2	$1.0000e^{+j0.8749}$
6	3.275	0.20	10.0	$1.0000e^{-j2.9486}$
7	3.775	0.24	1.0	$1.0000e^{+j0.2428}$
8	4.025	0.28	0.0	$1.0000e^{+j1.4916}$
9	4.525	0.26	0.0	$1.0000e^{+j1.4225}$
10	5.025	0.25	7.0	$1.0000e^{+j2.9057}$
11	5.525	0.29	6.0	$1.0000e^{+j2.8878}$
12	6.025	0.21	4.4	$1.0000e^{-j0.0151}$
13	6.775	0.29	0.0	$1.0000e^{-j1.3652}$
14	7.025	0.21	1.0	$1.0000e^{-j1.6289}$
15	7.275	0.25	-2.1	$1.0000e^{+j1.4025}$
16	7.775	0.21	3.0	$1.0000e^{+j2.5901}$
17	8.025	0.26	0.0	$1.0000e^{+j0.1691}$
18	8.525	0.20	0.0	$1.0000e^{-j1.3142}$
19	9.275	0.28	5.0	$0.5146e^{-j1.2172}$
20	9.775	0.28	4.7	$1.0000e^{-j1.9931}$
21	10.025	0.29	-8.9	$1.0000e^{-j1.1493}$
22	10.275	0.30	3.0	$1.0000e^{-j1.2696}$
23	10.525	0.25	3.2	$1.0000e^{-j1.5928}$
24	11.025	0.23	2.8	$1.0000e^{-j0.8409}$
25	11.275	0.21	2.9	$1.0000e^{-j0.4063}$
26	11.775	0.25	1.5	$1.0000e^{+j0.2300}$
27	12.025	0.26	0.7	$0.8833e^{-j2.9360}$
28	12.275	0.28	0.3	$1.0000e^{+j2.9279}$
29	13.275	0.21	0.0	$1.0000e^{-j0.7950}$
30	13.525	0.27	0.4	$1.0000e^{-j0.5133}$
31	14.025	0.25	-20.0	$0.6462e^{-j0.6720}$
32	14.775	0.22	0.8	$0.7048e^{+j2.6412}$

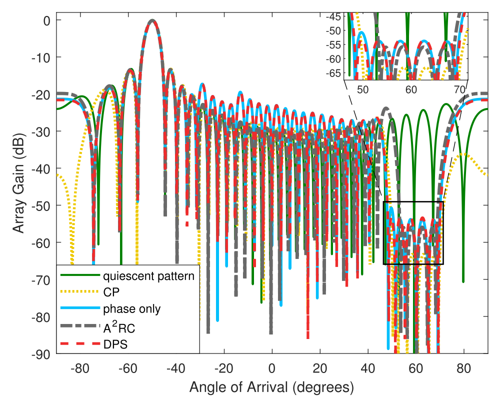

Fig. 5. Beampattern comparison for notch minimization with an ULA.

TABLE II

NOTCH LEVELS FOR NOTCH MINIMIZATION WITH A RANDOM ARRAY IN FIG. 7

	CP	phase only	A^{2}RC	DPS
notch level(dB)	-61.2064	-35.9983	-28.0262	-61.2064

The obtained weight vector for this part is shown in Table III. The mainloss is 3 dB, and from Fig. 7, we can see all the methods meet the requirement well at the beam axis. As shown in Table II, the notch level of the proposed method is -61.2064 dB, which is the same as that of the CP method. Moreover, the notch level of the proposed method is lower than that of the A^{2}RC method and the phase-only method, which are -28.0262 and -35.9983 dB. Thus, the validity of the proposed method has been verified again for notch minimization.

C. Beampattern on the Quantization of Phase Shifters

In practical applications, the phase shifters are controlled digitally and assume a finite number of values, depending on the quantization,

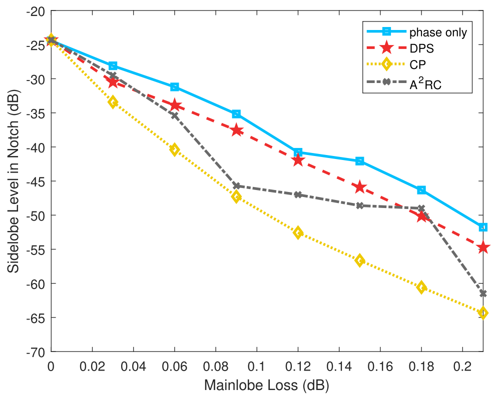

Fig. 6. Curves of the obtained notch level versus the permissible loss in mainlobe.

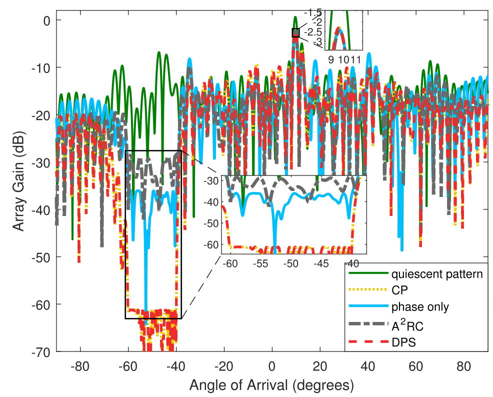

Fig. 7. Beampattern comparison for notch minimization with a random linear array.

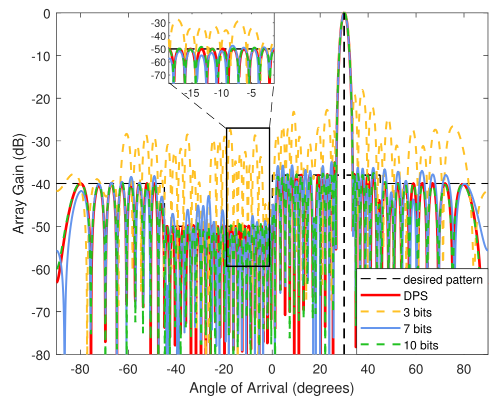

Fig. 8. Beampattern on the quantization of phase shifters.

TABLE III

OBTAINED WEIGHTINGS BY THE PROPOSED METHOD

n	w_n	$\angle w_{1,n}$(rad)	$\angle w_{2,n}$(rad)
1	$0.0176e^{+j0.9810}$	5.7110	2.5342
2	$0.0334e^{+j0.5220}$	5.2678	2.0594
3	$0.1248e^{-j2.5203}$	2.3172	5.2085
4	$0.2353e^{+j2.1284}$	0.7951	3.4617
5	$0.3357e^{-j2.7290}$	2.3257	4.7826
6	$0.1497e^{+j2.6900}$	1.2695	4.1105
7	$0.0927e^{-j1.4463}$	3.3590	0.0317
8	$0.4186e^{-j1.2116}$	3.9327	6.2105
9	$0.3312e^{-j1.8879}$	3.1621	5.6286
10	$0.0529e^{-j3.0575}$	1.7078	4.7436
11	$0.0529e^{-j1.8869}$	2.8784	5.9141
12	$0.1556e^{+j2.3558}$	0.9413	3.7740
13	$0.0644e^{+j1.5272}$	0.0208	3.0335
14	$0.0417e^{+j0.4317}$	5.1678	1.9429
15	$0.0280e^{-j0.6872}$	4.0531	0.8556
16	$0.2648e^{-j1.7470}$	3.2334	5.8390
17	$0.2532e^{+j0.1292}$	5.0976	1.4440
18	$0.3703e^{+j0.0349}$	5.1267	1.2264
19	$0.1949e^{-j0.5965}$	4.3120	0.7781
20	$0.2370e^{-j1.5158}$	3.4359	6.0988
21	$0.1416e^{+j1.5375}$	0.1088	2.9663
22	$0.1660e^{+j1.6387}$	0.2346	3.0427
23	$0.4149e^{+j0.5644}$	5.7046	1.7074
24	$0.3494e^{+j1.0891}$	6.1584	2.3030
25	$0.1929e^{-j0.2585}$	4.6480	1.1182
26	$0.1248e^{+j1.1351}$	5.9727	2.5808
27	$0.1403e^{-j2.4030}$	2.4502	5.3102
28	$0.1280e^{+j2.7873}$	1.3449	4.2297
29	$0.0492e^{+j0.7552}$	5.5169	2.2768
30	$0.1408e^{+j0.2220}$	5.0757	1.6516
31	$0.0532e^{+j0.1996}$	4.9653	1.7172
32	$0.0680e^{-j2.8641}$	1.9164	4.9218

which leads to errors. Thus, we need to consider whether the proposed method can synthesize the required beam shape after quantization.

For illustration, we consider an ULA with $N = 64$ elements. The spacing between two adjacent elements is half wavelength. We steer the beam to $\theta_0 = 30°$ and constrain all sidelobes. We divide the sidelobe region into four parts, which are $[-90°, -45°]$, $(-45°, 0°]$, $(0°, 29°) \bigcup (31°, 45°]$, and $(45°, 90°]$. Also, the upper bounds of the corresponding sidelobe levels are -40, -50, -38, and -40 dB.

The resulting beampatterns are shown in Fig. 8. We can see that the resulting beampattern by the proposed method meets all the constraints well. Moreover, with the improvement of quantization accuracy, the beampattern synthesized by the quantized weight vector is closer to the original beam pattern. When the quantization bit is 3, the quantized beampattern is very different from the original beampattern, except for the mainlobe region. With seven quantization bits, the quantized beampattern and the original beampattern are roughly the same, with some small differences. The quantized beampattern and the original beampattern are almost the same when the quantization bits reach 10. This implies that after quantification, the method proposed in this communication can synthesize a desired beampattern. Moreover, under the same quantization bits, the better performance can be obtained using the quantification methods in [18] and [19].

V. CONCLUSION

In this communication, we have proposed a beampattern synthesis method based on the DPS structure. In the DPS structure, since the modulus of weight vector can be changed in a continuous interval, we can turn the beampattern synthesis problems into convex problems, which can be solved by toolbox. Also, we have considered two different synthesis problems, which are mainlobe gain maximization and notch level minimization. We have performed simulations, and the numerical results have indicated that the proposed method is effective. Since the modulus of the weight vector is constrained, the proposed method may not perform as good as those methods that have no constraint on magnitudes. Nevertheless, the performance of the proposed method is satisfactory in most cases. As a future work, we will try to apply the DPS method to 2-D and other arrays.

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